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CONTROL SYSTEMS LABORATORY

RAPID SCAN

VERSUS

COHERENT DOPPLER RADAR FOR A.S.W.

Report R-51
February 1954
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COHERENT DOPPLER RADAR FOR A.S.W.

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I. Introduction

This report compares quantitatively the performance of coherent and rapid scan radar in their ability to detect targets in sea clutter. The analysis is made briefly and in rather simple mathematical form so as not to obscure the main arguments. The investigators feel that the available data and the approximations necessary do not warrant treating all the problems in minute detail. The comparison is made by first considering a conventional radar and then, with this as a reference, evaluating the improvement due to increasing the scanning speed and the improvement obtained by using coherent detection. These are compared and discussed.

In this analysis we assume that the sea return has a R. F. (or I. F.) spectrum of Gaussian shape and half power bandwidth Δf about the carrier and each of the sidebands, and that Δf is proportional to the carrier frequency. Measurements of sea return made by the Control Systems Laboratory at X-Band indicate that the clutter spectrum is Gaussian with a half power bandwidth of about 90 cycles. This number varies some with sea state and depression angles.*

* See CSL report R-37.

II. General Analysis

A. Conventional Radar

We start by considering a conventional pulse radar system which is to search an azimuth sector θ_s with an antenna having a square radiation pattern in azimuth of beamwidth θ_a. We also assume that the time for one sweep in azimuth is T and that the pulse repetition frequency is f_r. The time taken by the radar to sweep by a point in azimuth is therefore $\frac{T \ \theta_a}{\theta_s}$. However, the correlation time for the sea return is shown in Appendix A, to be approximately $\frac{1}{1.5 \ \Delta f}$ so that in effect the return per scan consists of only $1.5 \ \Delta f \ T \frac{\theta_a}{\theta_s}$ statistically independent samples. These independent samples are integrated by a combination of the radar screen and the observer.

B. Rapid Scan System

We next consider a system which scans more rapidly than the conventional radar but with the other system parameters unchanged. Let the time for each scan in the rapid scan system be $T_{rs} < T$. The total number of echoes received in time T from every patch of sea is independent of T_{rs}. However, if T_{rs} is made sufficiently small so that the time spent on each patch per scan is less than the correlation time of the sea return, or $\frac{T_{rs} \ \theta_a}{\theta_s} < \frac{1}{1.5 \ \Delta f}$ then an improvement is obtained in detectability. This is due to the increase in the number of statistically independent samples received in time T from $1.5 \ \Delta f \ T \frac{\theta_a}{\theta_s}$ to $\frac{T}{T_{rs}}$.

The minimum value of T_{rs} is limited by the requirement that the antenna scan slowly enough to transmit and receive an echo from every area in the scanned sector. This requires that at least two pulses be transmitted in the time it takes to sweep by a fixed point in the sector. Thus, the minimum value of T_{rs} is $\frac{2\,\theta_s}{f_r\,\theta_a}$. For this minimum value of T_{rs} the number of statistically independent samples from a given patch in time T is $\frac{T\,\theta_a\,f_r}{2\,\theta_s}$. This represents an increase over the number obtained by the conventional radar by a factor of $\frac{f_r}{3\Delta f}$.

The signal to clutter power required in the rapid scan radar for the same detectability as the conventional radar is reduced by a factor of approximately $\left[\frac{f_r}{3\Delta f}\right]^{3/4}$*.

These results indicate that it is advantageous for the rapid scan system to use as high a pulse repetition frequency as possible, thereby permitting a high scan speed. The usual limiting factor for the pulse repetition frequency is the unambiguous range requirement of the system so that adequate

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* The exponent 3/4 is approximate. The usual assumption is that the improvement due to integration is proportional to the square root of the number of integrations but strictly speaking this is justifiable only if the probability distributions involved are Gaussian which is not the case here. For further discussion of this point see J. I. Marcum "A Statistical Theory of Target Detection by Pulsed Radar", Rand Corporation RM 753, 754, (Figures 55 and 56 in RM 753 and Page 58 in RM 754).

time must be given for the transmitted pulse to return before the next one is generated. It is interesting to note though that even without this limitation there is an upper limit in the improvement obtained by increasing the scan speed. This upper limit is reached when the time between looks at each patch is smaller than the correlation time of the clutter and the looks are not independent. In the limit $T_{rs} \rightarrow 0$, the number of independent looks at each patch in time T is $1.5\ \Delta fT$. However, for this theoretical limit to be realized the antenna scan speed must be in the order of $1.5\ \Delta f$ scans per second and the pulse repetition frequency must be $3\ \Delta f\ \frac{\theta_s}{\theta_a}$. Both of these quantities are greatly beyond the practical limits of the system.

Figure 1 shows a sketch of the number of independent samples received from every patch in time T as a function of T_{rs}.

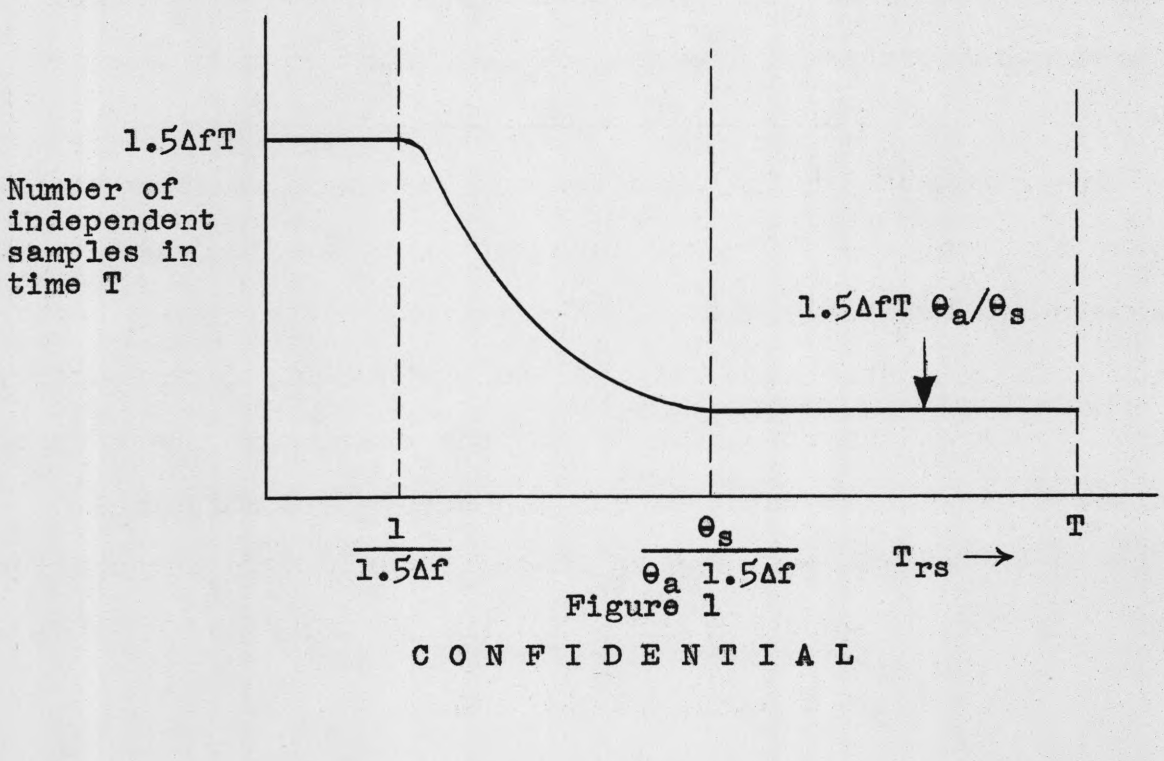

Figure 1

C. Coherent Radar

A coherent radar is capable of better performance than the conventional radar primarily because of its capability of resolving target velocities. We should, therefore, expect that the performance of the coherent radar as a snorkel detector depends greatly on the relative velocity of the radial component of the target with respect to the sea scatterers. A coherent radar may resolve the received spectrum from a patch to an accuracy equal to the reciprocal of the time it takes the antenna illumination to sweep by a point in the scanned sector. If we assume that the bandwidth (between 3db points) of each of the filters in the filter bank used to resolve target velocities is 'b' and the filters have a Gaussian transfer characteristic then we can calculate the clutter power passed by each filter. The fraction of the total clutter power passed by a filter whose center frequency is displaced from the center frequency of the clutter by an amount δ is shown in Appendix B to be

$$\frac{b/\Delta f}{\left[1 + (b/\Delta f)^2\right]^{1/2}} \quad \exp.\left[-2.77 \frac{(\delta/\Delta f)^2}{1 + (b/\Delta f)^2}\right]$$

The conventional radar will, in the time for one look at the target (i.e. $1/b$), integrate a total of $\Delta f/b$ independent samples. So that the essential difference between the coherent and conventional systems is that of pre and post detection integration. This matter has been discussed by others* and in the case where the clutter (or noise) is uniformly distributed over a band the two systems are, with present techniques, practically equivalent in their ability to detect targets. However, for the problem we are discussing this is not the case. The clutter spectrum is narrow and the signal may appear anywhere in the vicinity of the clutter. We feel therefore that if the signal were to appear at the center of the clutter spectrum (i.e. the target has zero radial component of velocity with respect to the sea scatterers) then this would be equivalent to the problem of a signal in a uniform noise spectrum and the coherent system would have little advantage to the conventional system. The advantage comes for signals of frequency different from the clutter center frequency. Then the gain may be given quantitatively by the ratio of clutter power passed by the filter which carries the signal to the one located at the center frequency of the clutter. This ratio is $\exp.\left[\frac{2.77\,(\delta/\Delta f)^2}{1+(b/\Delta f)^2}\right]$ or $\frac{12.0\,(\delta/\Delta f)^2}{1+(b/\Delta f)^2}$ db. Figure 2 shows the improvement factor as a function of 'b' for several values of target speed.

* See for example, "The Relative Sensitivities of Pre and Post Detection Integrators" by F. A. Rodgers Technical Memo Number 35 July, 1953, Lincoln Lab. M.I.T.

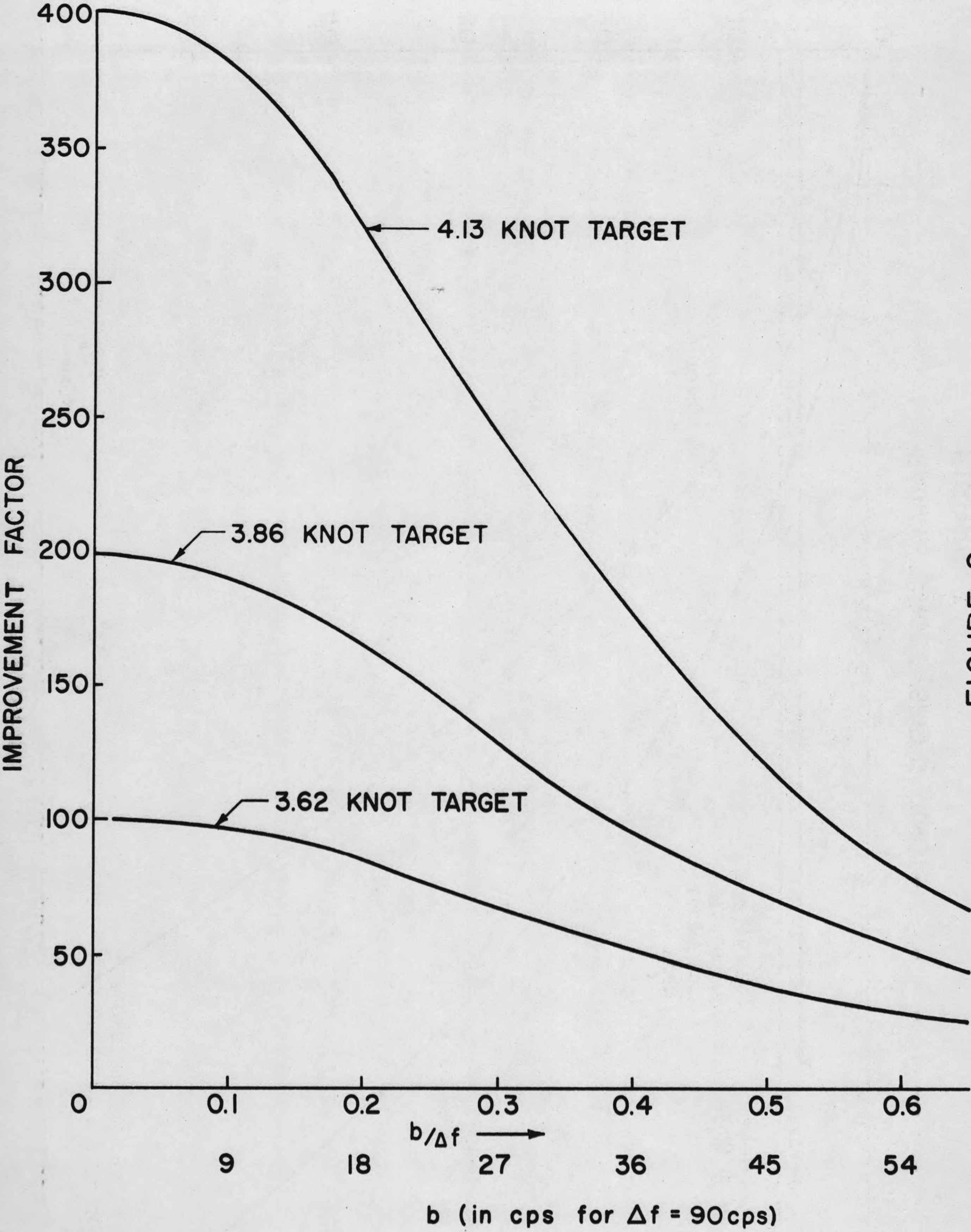
400
350
300
250
200
150
100
50
0
IMPROVEMENT FACTOR
4.13 KNOT TARGET
3.86 KNOT TARGET
3.62 KNOT TARGET
0.1
0.2
0.3
0.4
0.5
0.6
b/Δf
9
18
27
36
45
54
b (in cps for Δf = 90cps)

FIGURE 2

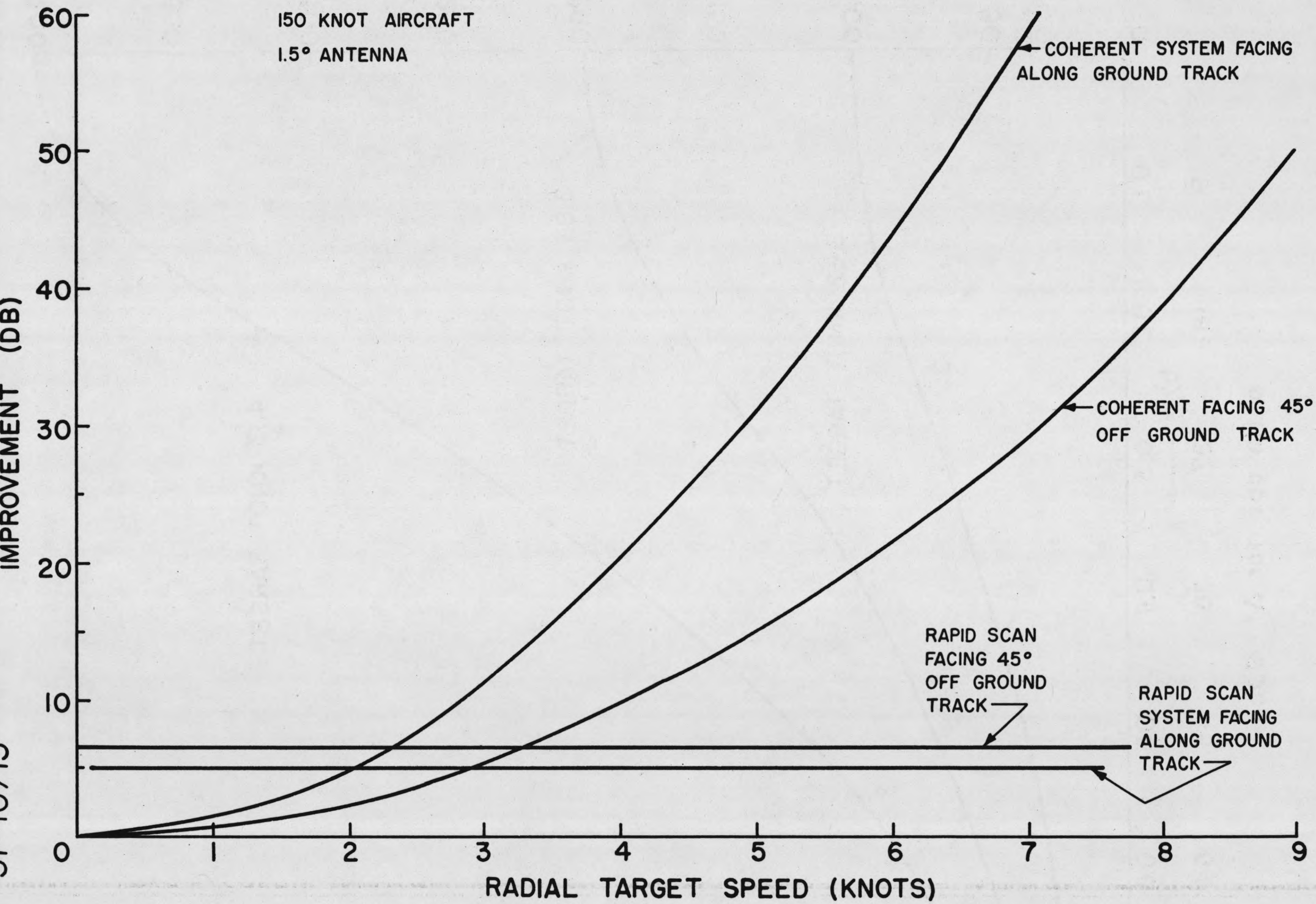

FIGURE 3

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III. Numerical Example

Consider an X-band radar intended for 40 mile range. This would require a prf of 2000 cps. The clutter bandwidth at X-band is 90 cycles. There is however, an additional broadening due to the variation in radial velocity of the clutter within an antenna beamwidth.* For a 1.5° antenna beamwidth and a 150 knot aircraft with the antenna facing 45° off the ground track, the spectrum broadens to about 130 cycles.

The Rapid Scan system improvement would therefore be $\left[\frac{2000}{3x90}\right]^{3/4} = 4.5$ or 6.5 db for the antenna facing along the ground track and $\left[\frac{2000}{3x130}\right]^{3/4} = 3.4$ or 5.33 db for the antenna facing 45° off the ground track. The Coherent Radar improvement is a function of target velocity as was shown previously. These results are plotted in Figure 3.

* See CSL Report R-38.

Correlation Time

We shall consider the correlation time T of a signal as being precisely $\int_{-\infty}^{+\infty} r(t') \, dt'$, where $r(t')$ is the normalized autocorrelation function of the signal. This definition is suggested by the work of Sherwin,* who has shown that linearly integrating a signal of finite power spectrum width for a duration long compared to $\int_{-\infty}^{+\infty} r(t') \, dt'$ is equivalent to adding independent samples of this signal at a rate of $\dfrac{1}{\int_{-\infty}^{+\infty} r(t') \, dt'}$.

If the signal is the output of a square law detector with an input which is narrow band noise of spectral density $G(f)$ centered at f_o plus a cw signal $S \cos 2\pi f_o t$, then we may evaluate T as follows:

Let $x(t)$ be the detector output. Then

$$r(t') = \frac{\overline{x(t)\, x(t + t')} - \overline{x}^2(t)}{\overline{x^2}(t) - \overline{x}^2(t)} .$$

Since x is proportional to the square of the envelope of the detector input, we may use the results of Sec 7.2** of "Threshold Signals" (Volume 24 of the Radiation Lab. Series) to obtain for $r(t')$ the equation $r(t') = \dfrac{\varphi(t')\, S^2 + \varphi^2(t')}{\sigma^2 (S^2 + \sigma^2)}$, where $\varphi(t') = \int_0^\infty G(f) \cos 2\pi (f - f_o)\, t' \, df$ and σ^2 = total noise power at the detector input $= \varphi(o)$. For $S >> \sigma$

$$T = \int_{-\infty}^{+\infty} r(t')\, dt' = \int_{-\infty}^{+\infty} \frac{\varphi(t')\, dt'}{\sigma^2} = \frac{G(f_o)}{\sigma^2}$$

which is by definition the reciprocal of the noise bandwidth B of the input.

* See CSL Report R-42.

** See Equation 17.

We have tabulated below the result for T for a variety of pre-detector noise bandshapes. In calculating these we have used Parseval's Theorem that $\int_{-\infty}^{\infty} \varphi^2(t') \, dt' = \int_{0}^{\infty} G(f) \, df$.

Power Spectrum of Predetector Noise	Ratio of noise bandwidth β to half power bandwidth Δf	Correlation time T	T [S=o]
$G(f) = \begin{cases} 1 & f_o - \beta/2 < f < f_o + \beta/2 \\ o & \text{elsewhere} \end{cases}$	$\beta/\Delta f = 1$	$T = 1/\beta$	$T = 1/\beta$
$G(f) = \exp\left[-\pi/\beta^2 (f-f_o)^2\right]$	$\beta/\Delta f = 1.07$	$T = 1/\beta \dfrac{\frac{S^2}{2\sigma^2} + \frac{1}{2\sqrt{2}}}{\frac{3^2}{2\sigma^2} + \frac{1}{2}}$	$T = 1/\sqrt{2}\beta$
$G(f) = \dfrac{1}{1 + \pi^2/\beta^2 (f-f_o)^2}$	$\beta/\Delta f = \pi/2$	$T = 1/\beta \dfrac{\frac{S^2}{2\sigma^2} + \frac{1}{4}}{\frac{S^2}{2\sigma^2} + \frac{1}{2}}$	$T = 1/2\beta$

In this appendix we shall simply calculate the fraction of clutter power passed by a Gaussian band pass filter whose center frequency is displaced from that of the clutter by an amount δ.

Let the clutter spectrum be given by $G(f) = \exp.\left[-\frac{\pi(f-f_0)^2}{\beta_1^2}\right]$ where β_1 = equivalent square bandwidth of clutter so that the total clutter power is β_1.

If the filter transfer characteristic is

$$Y(f) = \exp.\left[-\frac{\pi(f-f_0-\delta)^2}{2\,\beta_2^2}\right],$$

where β_2 is the noise bandwidth of the filter, then the total clutter power output after filtering is

$$\int_{0}^{\infty} G(f)\,\left|Y(f)\right|^2\,df = \int_{-\infty}^{+\infty} \exp.\left[-\frac{\pi(f-f_0-\delta)^2}{\beta_1^2}\right] \exp.\left[\frac{\pi(f-f_0)^2}{\beta_2^2}\right] df$$

$$= \frac{\beta_1\,\beta_2}{\sqrt{\beta_1^2 + \beta_2^2}}\ \exp.\left[-\frac{\pi\,\delta^2}{\beta_1^2 + \beta_2^2}\right]$$

and the fraction of the total power passed by the filter is $\frac{\beta_2}{\sqrt{\beta_1^2 + \beta_2^2}}\ \exp.\left[-\frac{\pi(\delta/\beta_1)^2}{1 + (\beta_2/\beta_1)^2}\right]$. If we formulate this in terms of half power bandwidths then the fraction of clutter power passed is $\frac{b/\Delta f}{\sqrt{1 + (b/\Delta f)^2}}\ \exp.\left[-\frac{2.77\,(\delta/\Delta f)^2}{1 + (b/\Delta f)^2}\right]$ where Δf is the half power bandwidth of the clutter and b is the half power bandwidth of the filter.